DOCUMENT RESUME

ED 022 692 SE 005 042

By-Fehr, Howard F.

SECONDARY SCHOOL MATHEMATICS CURRICULUM IMPROVEMENT STUDY--SUMMARY OF OBJECTIVES AND ACTIVITY, SEPTEMBER, 1965 TO JANUARY, 1968.

Columbia Univ., New York, N.Y. Teachers College.
Pub Date 68
Note-14p.
EDRS Price MF-$0.25 HC-$0.64
Descriptors-COURSE CONTENT, *CURRICULUM, *CURRICULUM DEVELOPMENT, *EDUCATIONAL OBJECTIVES, *MATHEMATICS, OBJECTIVES, *SECONDARY SCHOOL MATHEMATICS
Identifiers-Secondary School Mathematics Curriculum Improvement Study

Reported is an experimental study by the Secondary School Mathematics Curriculum Improvement Study (SSMCIS) whose objective is the construction of a unified school mathematics curriculum for grades 7 through 12. More specifically, SSMCIS is concerned with the reconstruction of the entire mathematics curriculum. It attempts to eliminate the barriers separating the several branches of mathematics and to unify the subject through such fundamental concepts as sets, operations, mappings, and relations. Included is an outline and brief description of the materials for the first two courses in the sequence. Behavioral objectives of the materials include (1) recall definitions, notations, operations, and concepts, (2) manipulate and calculate efficiently, (3) interpret symbolic data or processes, (4) communicate mathematical ideas, (5) apply concepts to a purely mathematical situation--solve problems, (6) apply concepts to problems in other situations--solve work problems, (7) transfer learning to a new situation in mathematics, and (8) construct or follow a mathematical argument. At present the program is designed for the more capable students (roughly those in the top 20 per cent of their class with respect to mathematical ability). (RP)

SECONDARY SCHOOL MATHEMATICS CURRICULUM IMPROVEMENT STUDY

Summary of

OBJECTIVES AND ACTIVITY

September, 1965 to January, 1968

Teachers College, Columbia University

New York, New York 10027

Howard F. Fehr, Project Director

ED022692

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
OFFICE OF EDUCATION

THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL OFFICE OF EDUCATION POSITION OR POLICY.

SE 005 042

ACKNOWLEDGMENTS

Planning, writing, and teaching of SSMCIS Courses. was done with the cooperation of the schools and teachers in the Metropolitan New York Area and the following consultants:

Gustav Choquet, Université de Paris, France
Ray Cleveland, University of Calgary, Canada
John Downes, Emory University
Howard F. Fehr, Teachers College, Columbia University (Director)
James Fey, Teachers College, Columbia University
Abraham Glicksman, Bronx High School of Science, New York
Vincent Haag, Franklin and Marshall College
Thomas Hill, University of Oklahoma
Peter Hilton, Cornell University
Julius H. Hlavaty, National Council of Teachers of Mathematics
Meyer Jordan, City University of New York
Burt Kaufman, Southern Illinois University
Erik Kristensen, Aarhus University, Denmark
Howard Levi, City University of New York
Edgar Ray Lorch, Columbia University
Lennart Råde, Chalmers Institute of Technology, Sweden
Harry Ruderman, Hunter College High School New York
Harry Sitomer, C. W. Post College
Hans-Georg Steiner, University of Münster, Germany
Marshall H. Stone, University of Chicago
H. Laverne Thomas, State University College at Oneonta, New York
Albert W. Tucker, Princeton University
Bruce Vogeli, Teachers College, Columbia University

Introduction

During the past decade the United States has been engaged in revising the elementary and secondary school mathematics curriculum--primarily by up-dating the existing traditional curriculum. Modest recommendations of the Commission on Mathematics have been largely accepted by curriculum and syllabus bodies and by writers of commercially produced textbooks. Implementation of this program by the SMSG has had wide acceptance and massive experimental use throughout the country.

Throughout all of our reform movements the traditional division of mathematics instruction into separate years of arithmetic, algebra, and geometry has been maintained. Beyond introduction of new concepts, little has been gained in bringing more advanced study into the high school through more efficient methods of organizing the subject matter. Bolder and more radical recommendations for the improvement of secondary school education in mathematics have been made both in this country, notably by the UICSM, and in Europe, notably in Belgium, Switzerland, and Denmark.

What has been called for is reconstruction of the entire curriculum from a global point of view--one which eliminates the barriers separating the several branches of mathematics and unifies the subject through its general concepts (sets, operations, mappings, and relations) and builds the fundamental structures of the number systems, algebra, and geometry (groups, rings, fields, and vector spaces). The efficiency gained by such organization should permit introduction into the high school program of much that was previously considered undergraduate mathematics.

In September 1965, the Commissioner of Education, Department of Health, Education, and Welfare, Office of Education, approved for support for a period of 18 months the Secondary School Mathematics Curriculum Improvement Study (SSMCIS), an experimental study whose objective would be the construction of a unified school mathematics curriculum for grades seven through twelve. This is a report of the activities and findings of the SSMCIS from its inception

Planning the 7-12 Program

Long range planning of the proposed six year study was begun with a two day meeting of chief consultants in November 1965. The participants at this meeting outlined procedures for subsequent syllabus conferences, writing of the experimental textbooks, and evaluation of teaching in pilot classes.

In June 1966 a group of eighteen leading United States and European mathematicians and educators met for 20 days to outline the scope and sequence of a six year unified secondary school mathematics program. The first half of the conference was devoted to producing a complete flow charted analysis of the proposed course. Then topics planned for the seventh grade were expanded in working papers which outlined the mathematical content of each textbook chapter and made specific suggestions for writing and teaching these ideas.

Writing of Course I

During July and August 1966, a team of eight mathematical educators wrote the textbook for Course I, using the syllabus produced in June as a guide. Each textbook chapter was written by one writer, reproduced for review by the other writers and consulting mathematicians, and then rewritten, incorporating the reviewers' suggestions. Teachers guides and solutions to exercises were written for each chapter. These notes, mimeographed and distributed to teachers of experimental classes, included discussions of fundamental mathematical ideas underlying each chapter, hints for possible class activity to accompany reading of the text, and suggested time allotment to the various topics.

The Course I textbook (Experimental Edition) contained 16 chapters and was published in three volumes. Chapter titles and brief descriptions appear below. A more detailed description, including chapter subheadings, is given as Appendix A of this report.

Chapter	Title	Description
0	Planning a Mathematical Process	Introduction to flow charting algorithms.
1	Finite Number Systems	Study of properties of modular arithmetic systems.
2	Sets and Operations	Introduction to binary operations.
3	Mathematical Mappings	Introduction to concept of mapping.

Chapter	Title	Description
4	The Integers	Addition of Integers.
5	Probability and Statistics	First concepts of probability and descriptive statistics.
6.	Multiplication of Integers	Definition and properties of $(Z,\cdot)$.
7	Lattice Points in the Plane and Mappings on Z x Z	Lattice representation of pairs of integers.
8	Sets and Relations	Set notation and properties of relations.
9	Transformations of the Plane and Orientations in the Plane	Line reflections, point symmetries, rotations, and translations.
10	Segments, Angles, and Isometries	Measure of segments and angles and preservation under certain mappings.
11	Elementary Number Theory	Divisibility, primes, and the Euclidean algorithm.
12	The Rational Numbers	Addition, multiplication, and order of rational numbers.
13	Mass Point Geometry	A small deductive system involving mass points.
14	Some Applications of the Rational Numbers	Dilations, ratio and proportion, percent, and translations.
15	Incidence Geometry	A small axiomatic affine geometry.

Education of Teachers

During the June 1966 conference a special program of study was arranged to prepare all the teachers of experimental classes to teach Course I. Then each of the experimental teachers studied four hours daily for thirty days during the Teachers College 1966 Summer Session. The instruction covered fundamental mathematical concepts underlying the unified mathematics program and contemporary methods of teaching standard and new mathematical topics.

The program included the courses:

TX 4351--Modern Mathematical Structures

Theory of sets; groups, rings, ordered fields, isomorphism; affine space; euclidean space; real numbers; vector spaces; numerical calculus; statistics.

taught by Mr. Burt Kaufman of Southern Illinois University, and

TX 4406--Teaching Contemporary Junior High School Mathematics

Emphasis on teaching mathematics as a unified branch of knowledge. Teaching: set theory, mapping, relations, and functions; structure of number systems; groups, rings, and field properties; algebraic structure; vectors; translations, reflections, rotations; symmetries; plane geometry. Experimental programs and evaluation of mathematical learning.

taught by Dr. Julius H. Hlavaty, a chief consultant to the project.

The following is a list of the teachers and the schools in which they taught the experimental classes during the 1966-67 school year:

Carbondale, Illinois, University High School
Mr. Dave Masters

Elmont, New York, Alva Stanforth Junior High School
Mr. Alexander Imre
Mr. Edward Keenan

Fort Washington, Pennsylvania, Germantown Academy
Mr. Ronald Craig
Mr. Wirt Thompson

Glen Rock, New Jersey, Glen Rock Junior High School
Mr. James Law
Mr. Neil McDermott

Leonia, New Jersey, Leonia High School
Miss Christine McGoey
Mr. Kenneth McGown

New York, N.Y., Hunter College High School
Mr. Richard Klutch
Miss Ruth Morgan

Port Chester, New York, Ridge Street School
Miss Riva Machlin
Mr. Thomas Reistetter

Teaneck, New Jersey, Benjamin Franklin Junior High School
Mrs. Annabelle Cohen
Mr. Otto Krupp

Thomas Jefferson Junior High School
Mr. Franklin Armour
Miss Louise Fischer

Westport, Connecticut, Bedford Junior High School
Mr. James Detweiler
Mr. Ray Walch

Each teacher was taken through selected chapters of the following books:

1) T. J. Fletcher, ed., Some Lessons in Mathematics, Cambridge Press.

2) D. E. Mansfield and M. Bruckheimer, Major Topics in Modern Mathematics, Harcourt, Brace, World.

3) G. Papy, Mathématiques Moderne I, Didier.

4) Mimeographed version of the experimental Course I textbook.

All teachers showed intense interest and cooperated splendidly in acquiring the spirit and content of the proposed new curriculum.

Teaching Course I

Nine junior high schools in the Metropolitan New York area and one in Carbondale, Illinois, were selected to participate in the experimental teaching of Course I. In each school two teachers who had received special summer instruction were assigned to teach a single pilot class. Eight of these classes were at the seventh grade level and two at the eighth grade level--involving a total of 350 students.

The SSMCIS program is at present designed for capable students roughly those in the top 20% of their class with respect to mathematical ability. With this population in mind, pilot classes were selected by the participating schools using prior mathematics achievement and scores on aptitude tests as the main criteria.

For the first year large classes (35-40 students) were encouraged so that a gradual drop out would enable a class of sufficient size to complete the sixth year of the program. However, at the end of the first year very few pupils are leaving the program and all classes, except that at Glen Rock, where administrative problems make it impossible to continue, will move ahead into Course II during the 1967-68 school year. Teachers and students alike have found the mathematics intellectually stimulating, interesting, and enjoyable. In fact, several students who have been forced to leave the program because of family change of residence have asked to be allowed to continue studying the SSMCIS textbooks on their own.

Because the teachers of pilot classes were working as a team in the experimental class, they were often able to help each other with difficulties that arose in understanding or teaching the new material. Having had this year of team teaching experience, the teachers are now prepared to teach Course I individually. The revised Course I text will therefore be used in approximately 20 new seventh grade classes during the 1967-68 school year.

During the School year, the director and project staff members made frequent personal visits to observe the experimental teaching. Each class was observed at least four times. Visits to these schools included discussion with the teachers and administrators concerning progress and problems with the experimental course.

The teachers were further assisted by four full day Saturday meetings at Teachers College where teaching problems were reviewed with selected consultants and the project director. At these meetings many teaching difficulties were resolved and valuable criticisms of the textbook were gathered.

Evaluation of Course I

The six year mathematics program, of which Course I is only the first part, introduces many new concepts into the secondary school mathematics curriculum and integrates both standard and new topics in a global organization not characteristic of existing programs. Student achievement in such a program cannot adequately be measured using conventional standardized tests. For this reason, student learning was tested by three extramural examinations, constructed by the project staff.

To guide construction of these and future measurement instruments, the Course I textbook was analyzed to produce a taxonomy of cognitive objectives. This taxonomy delineated goals of instruction in terms of subject matter and related behaviors. The framework for this analysis is illustrated in the following tables.

TABLE I

SCHEME FOR TAXONOMY OF OBJECTIVES

Mathematical Objectives

Structures: Arithmetic and Algebra Geometry Probability

Fundamental Concepts: Sets Operations Relations Mappings Logic

Behavioral Objectives

I. Ability to recall definitions, notations, operations, concepts.

II. Ability to manipulate and calculate efficiently.

III. Ability to interpret symbolic data or processes.

IV. Ability to communicate mathematical ideas.

V. Ability to apply concept to a purely mathematical situation--solve problems.

VI. Ability to apply concept to problems in other situations--solve word problems.

VII. Ability to transfer learning to a new situation in mathematics.

VIII. Ability to construct or follow a mathematical argument.

Of course not all these categories apply to each subject matter topic, but the goals were checked against subject matter in a two-way cellular chart similar to the following:

CONTENT	BEHAVIOR I Ability to recall definitions, notations, operations concepts.	II Ability to manipulate and calculate efficiently.	III Ability to interpret symbolic data or process.	IV Ability to communicate mathematical ideas.	V Ability to apply concept to a purely mathematical situation. Solve problems.	VI Ability to apply concept to problemsin other situations. Solve word problems.	VII Ability to transfer learning to a new situation in mathematics.	VIII Ability to construct or follow a mathematical argument. (Proof)
Sets	X	X	X	X	X	X	X	X
A. Notation								
1.Inclusion ⊂	X		X	X	X			
2.Membership∈	X		X	X	X			
3.Empty ∅	X		X	X				
4.Ven Diagrams	X		X	X	X			
B. Cardinality								
1.Finite	X							
2.Infinite	X							
C. Set Equality								
1.Subsets	X	X						
2.Disjoint Sets	X							
D. Universal Sets Partitions	X	X	X		X	X		
E.Set Operation	X							
1.Union	X	X	X	X	X		X	X
2Intersection	X	X	X	X	X		X	X
3Complement		X	X	X	X		X	X

TABLE II
SAMPLE PAGE FROM TAXONOMY
OF OBJECTIVES

The three examinations, constructed from this taxonomy were designed to measure learning of the important new concepts in Course I such as operational system, mapping, and geometric transformation as well as standard seventh and eighth grade topics which remain a part of the new course. Copies of these tests, administered in November, February, and May, appear as Appendix B at the end of this report.

Although achievement on standardized traditional mathematics tests was not accepted as a measure of the success of the experimental program, it was of interest to determine whether or not study in the experimental Course I affected learning of traditional topics. To accomplish this objective all students were administered the Sequential Test of Educational Progress--Mathematics, Form 3A--in September 1966 and again in September 1967.

These test results clearly show that students in the project classes suffered no decline in mathematical skills when compared with students studying more traditional programs. Moreover, the achievement of these students on the project tests shows that they were learning to work with many new and powerful mathematical tools not a part of the traditional mathematics fare of seventh graders.

Item error analysis of the project test papers provided insight into the success of particular aspects of Course I instruction. The complete data available from these and other measures of student aptitude and achievement will be analyzed statistically during the year 1967-68.

Revision of Course I

Throughout the 1966-67 school year the project staff gathered detailed reactions to Course I from consultants and teachers, test results, and observations of the experimental teaching. These findings suggested the following revision of Course I.

1. The chapter on flow charting mathematical processes should be rewritten to emphasize graphic representation of algorithms rather than review and examination of the computational procedures of whole number arithmetic. In revised form this chapter would then be more appropriate as a summarizing chapter than an introductory one.

2. The chapter on addition of integers should be rewritten from a different point of view since the isomorphism concept will be removed from Course I and placed in Course II.

3. The chapter on mappings should be simplified and shortened by omitting the work with central and parallel projections.

4. The chapter on rational numbers should be rewritten in a style consistent with the new approach to the integers.

5. The concept of orientation should be transferred from the chapter on transformation in the plane to Course II.

6. The chapters on mass points and affine geometry should be expanded and made part of Course II.

Some of these revisions began in the spring of 1967; the remainder were left for the summer writing group.

This revised Course I textbook is being pilot tested again in the same schools during the 1967-68 school year. However, this year each teacher teaches a class of seventh graders individually. Therefore, the new seventh grade experimental population will include over 20 classes and 700 students. The work of this year will be carefully evaluated and in July, 1968 a final revised edition of Course I will be placed in the public domain.

Planning for Course II

A proposal for support of the continuation of the experimental curriculum study for a period of 18 months was presented to the Office of Education in January, 1967. On June 14, 1967 the proposal was approved, and initial financing of $112,000 was allocated for the period June 15, 1967 to January 31, 1968. Financing of the remaining 11 months of approved project activity was negotiated in fall of 1967.

The June 1966 syllabus conference prepared a tentative list of chapters for Course II. However, the order of these chapters, details for writing the textbook, and modifications due to the experimental teaching of Course I were planned in the spring of 1967.

During the summer of 1967 the first course was revised to reflect experience of the first year of experimental teaching. Then, following procedures similar to those used in developing the original first course, a new second course was written (for eighth grade) and teachers were given further preparation. This new course is now being taught in ten experimental schools and on the basis of this experimentation will be revised in the summer of 1968.

Outline of Topics -- SSMCIS Course II

Chapter I: Mass Points --	A small deductive system studying mass points on a line, in a plane, and in space.
Chapter II: Structures on Sets of Sets --	Equipotent sets, power set, characteristic function, lattice diagrams and partial ordering, measures on sets of sets, operations with complexes.
Chapter III: Groups --	Definition and examples of groups, permutations, some group theorems, isomorphism.
Chapter IV: Introduction to Axiomatic Affine Geometry -	The axioms of an affine geometry, consequences of the axioms, models, parallel lines, parallel projection, vectors.
Chapter V: Descriptive Statistics --	Graphical presentation for statistical data, range, quartile, median, mode, mean, variance, standard deviation.
Chapter VI Fields and Real numbers --	Definition and examples of a field, some field theorems, equations in a field, order in a field, $\sqrt{2}$, introduction to the real numbers.
Chapter VII Coordinate Geometry --	Line coordinate systems; segments, rays, and midpoints; parallel projection and plane coordinate systems; equation for a line; intersections of lines; triangles and quadrilaterals; pythagorean property and rectangular plane coordinate systems.
Chapter VIII Real Functions	- One-to-one and onto functions, graphing, composition, inverses, addition and multiplication of real functions, $\cdot\sqrt{}$ function.
Chapter IX Transformations in the Plane	- Isometries--Reflection in a line, translation, rotations and half-turns, glide reflections, group of isometries, congruence, dilations and similarities.

Chapter X Combinatorics -- Permutations, counting principle, number of subsets of r elements drawn from an n element set, Pascal's triangle, binomial theorem.

Chapter XI Space Geometry - Parallel, incident, and perpendicular lines and planes in space; transformations in space; space coordinate systems and the distance formula.

Chapter XII Introduction to Trigonometry -- Tangent, sine, and cosine functions; law of cosines, applications.

Chapter XIII Length, Area, and Volume - Lengths of line segments, circumference of circles, areas of planar regions, volumes of rectangular solids.

Materials Available

While both courses are being tested again this year in metropolitan area schools, a limited number of copies of the textbooks are available for examination by people interested in the school mathematics program.

Materials available for purchase are described below. To order books, all requests should be accompanied by check payable to Teachers College. The prices cited include cost of packing and mailing. Send orders to

SSMEIS, Box 120
Teachers College, Columbia University
525 West 120th Street
New York, N. Y. 10027

Course I, Part I (Revised Edition), 365 pp. $2.00

Course I, Part II (Revised Edition), 420 pp. $2.00

Course II, Part I (Experimental Edition), 335 pp. $2.00

Course II, Part II (Experimental Edition), 375 pp. $2.00